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Navlakhi's

Mechanics

Chapter 1: Resolution of Forces

Solutions

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POWER

Q2.1 $P = 18N$, $Q = 35N$, $\alpha = 45^\circ$

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}}$$

$$= \sqrt{(18)^2 + (35)^2 + 2(18)(35)\cos 45^\circ}$$

$R = 49.4N$

$$\boxed{\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}}$$

$$\tan\theta = \frac{35\sin 45^\circ}{18 + 35\cos 45^\circ}$$

$\tan\theta = 0.5789$

$\therefore \theta = 30.07^\circ$.

Q2.2. Let the forces be P and Q.

$\therefore P + Q = 20$ ——— (I)

$R = 12N$, $\theta = 90°$, P is smaller force.

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}}$$

$\therefore 12 = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

$\therefore 2PQ\cos\alpha + P^2 + Q^2 = 144$ ——— (II)

$$\boxed{\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}}$$

$\tan 90° = \dfrac{Q\sin\alpha}{P + Q\cos\alpha} = \infty$

$\therefore P + Q\cos\alpha = 0$ ——— (III)

Squaring (III) $\therefore P^2 + 2PQ\cos\alpha + Q^2\cos^2\alpha = 0$ — (IV)

(II) − (IV) $\therefore Q^2 - Q^2\cos^2\alpha = 144$

$\therefore Q^2(\sin^2\alpha) = 144$ $\therefore Q\sin\alpha = 12$ $\therefore Q = \dfrac{12}{\sin\alpha}$ — (V)

Subs. (V) in (I) and (III)

$\therefore P + \dfrac{12}{\sin\alpha} = 20$ and $P + \dfrac{12}{\tan\alpha} = 0$

$\therefore \dfrac{-12}{\sin\alpha} + \dfrac{12}{\tan\alpha} = 20$ $\therefore 12\left[\dfrac{1-\cos\alpha}{\sin\alpha}\right] = 20$

$\therefore \dfrac{2\sin^2\alpha/2}{2\sin\alpha/2\cos\alpha/2} = \dfrac{20}{12}$ $\therefore \tan\dfrac{\alpha}{2} = \dfrac{5}{3}$ $\therefore \alpha = 118.07°$

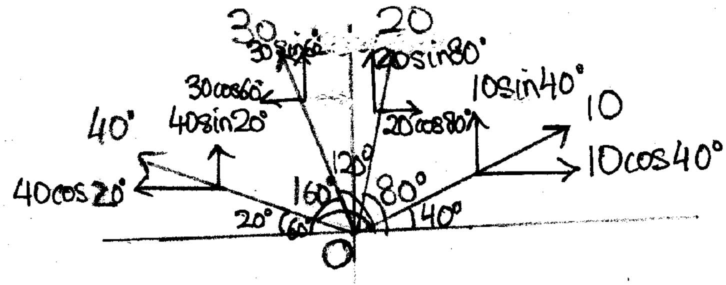
Subs. α in (V)

$\therefore \underline{Q} = \dfrac{12}{\sin 118.07} = \underline{13.6\,N}$

From (I)

$\underline{P = 6.4N}$

Q2.3



$$\Sigma F_x = 10\cos 40° + 20\cos 80° - 30\cos 60° - 40\cos 20°$$

$$= -41.454\,N$$

$$\Sigma F_y = 10\sin 40° + 20\sin 80° + 30\sin 60° + 40\sin 20°$$

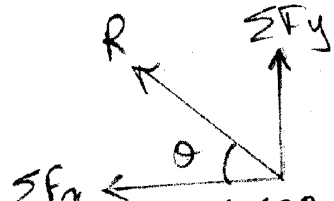
$$= 65.786\,N$$

$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$

$$= \sqrt{(-41.454)^2 + (65.786)^2}$$

$$= 77.76\,N$$

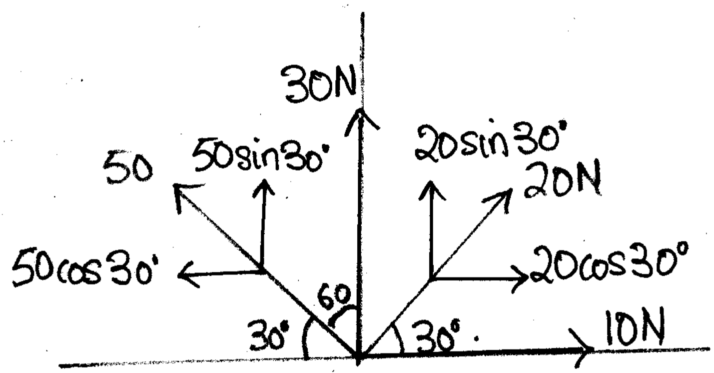
$$\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}} = \frac{65.786}{41.454}$$



$$\therefore \theta = 57.78°$$

Resultant force is $\underline{R = 77.76\,N}$ at $\underline{\theta = 57.78°}$ in the second quadrant.

Q2.4



$$\Sigma F_x = 20\cos 30^\circ - 50\cos 30^\circ + 10$$

$$= -15.98\ N$$

$$\Sigma F_y = 20\sin 30^\circ + 30 + 50\sin 30^\circ$$

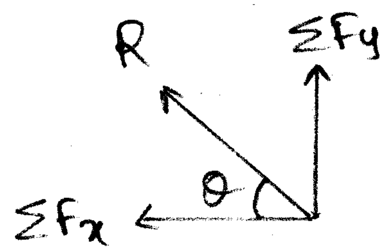
$$= 65\ N$$

$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$

$$= 66.94\ N$$

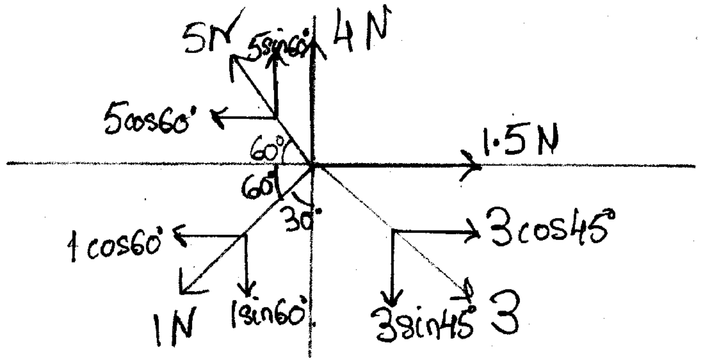
$$\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}} = \frac{65}{15.98}$$

$$\therefore \theta = 76.19^\circ$$



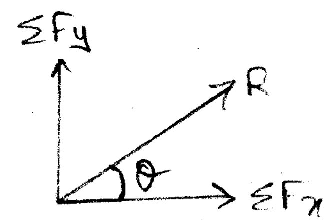
Resultant force $R = 66.94\ N$ at $\theta = 76.19^\circ$ in second quadrant.

Q2.5



$$\Sigma F_x = 1.5 - 5\cos60° - 1\cos60° + 3\cos45°$$
$$= 0.6213\ N$$
$$\Sigma F_y = 4 + 5\sin60° - 1\sin60° - 3\sin45°$$
$$= 5.343\ N$$

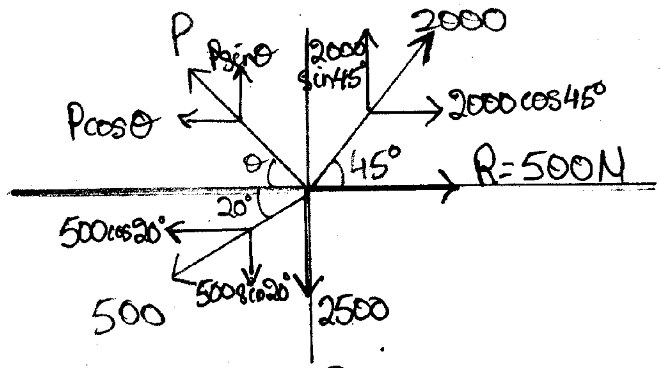
$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$
$$= \sqrt{(0.6213)^2 + (5.343)^2}$$
$$\therefore R = 5.38\ N$$



$$\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}} = \frac{5.343}{0.6213}$$
$$\therefore \theta = 83.37°$$

$\therefore$ Resultant force is $\underline{R = 5.38\ N}$ at $\underline{\theta = 83.37°}$ in first quadrant

Q.2.6



$$\Sigma F_x = 2000\cos 45° - P\cos\theta - 500\cos 20°$$
$$= 944.37 - P\cos\theta$$

$$\Sigma F_y = 2000\sin 45° + P\sin\theta - 500\sin 20° - 2500$$
$$= -1256.8 + P\sin\theta$$

$\therefore$ Resultant is horizontal

$\therefore$ $\boxed{\Sigma F_y = 0}$

$\therefore$ $P\sin\theta = 1256.8$ — (I)

Also, $\boxed{R = \Sigma F_x}$

$\therefore$ $500 = 944.37 - P\cos\theta$

$\therefore$ $P\cos\theta = 444.37$ — (II)

(I) ÷ (II) $\tan\theta = 2.829$

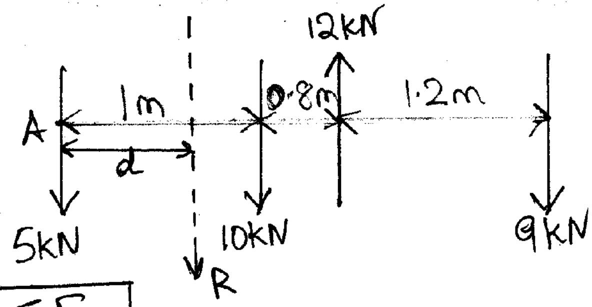
$\therefore$ $\cancel{\tan}\theta = 70.53°$

$\therefore$ $P\sin\theta = 1256.8$ {from (I)}

$\therefore$ $P = 1333.03N$

$\therefore$ Force <u>P = 1333.03N</u> at <u>$\theta = 70.53°$</u> in second quadrant.

Q2.7



$\boxed{R = \Sigma F_y}$

$= -5 - 10 + 12 - 9 = -12kN$

$\therefore R = 12kN (\downarrow)$

By Varignon's theorem, considering anti-clockwise moments as positive about point A & considering R to right of A

$\boxed{\Sigma M_A^F = M_A^R}$

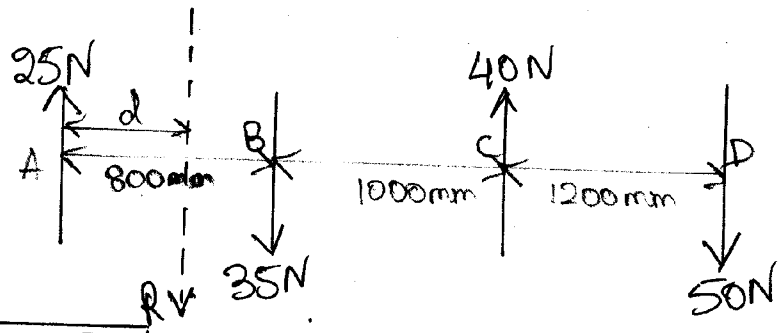
$\therefore (-10 \times 1) + (12 \times 1.8) - (9 \times 3) = -(R \times d)$

$\therefore -15.4 = -12 \times d$

$\therefore d = 1.283m.$

$\therefore$ Resultant force is 12kN ($\downarrow$) at a perpendicular distance of 1.283m to the right of A.

Q2-8



$$\boxed{R = \Sigma F_y}$$

$$= 25 - 35 + 40 - 50$$

$$= -20\,N$$

$\therefore R = 20N\ (\downarrow)$

By Varignon's theorem, considering anti-clockwise moments as positive about A and considering resultant (R) to the right of A

$$\boxed{\Sigma M_A^F = M_A^R}$$

$$(-35 \times 800) + (40 \times 1{,}800) - (50 \times 3000) = -R \times d$$

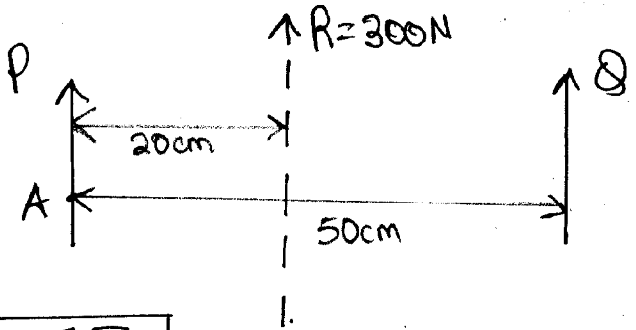
$$-1{,}06{,}000 = -20 \times d$$

$\therefore d = 5{,}300\,mm.$

$\therefore$ Resultant force is <u>20N (↓)</u> at a perpendicular distance of <u>5,300mm</u> to the right of A.

POWER

Q2.9



$$\boxed{R = \Sigma F_y}$$

$\therefore P + Q = 300$ ——— (I)

By Varignon's theorem, considering anticlockwise moments as positive about A and considering resultant (R) to the right of A

$$\boxed{\Sigma M_A^F = M_A^R}$$

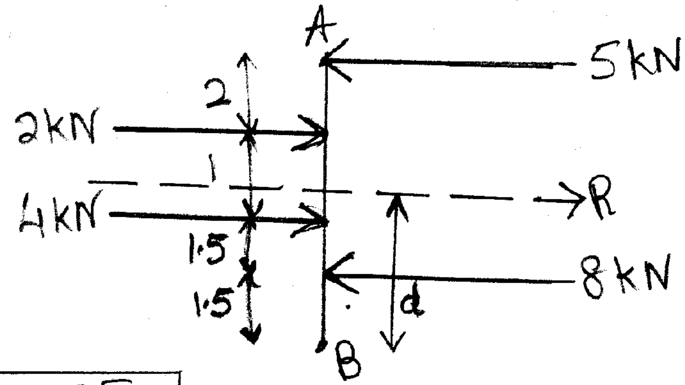
$Q \times 50 = R \times 20$

$\therefore \underline{Q} = \dfrac{300 \times 20}{50} = \underline{120N}\ (\uparrow)$

$\because P + Q = 300$ [from (I)]

$\therefore \underline{P = 180\,N}\ (\uparrow)$

Q2.10



$$\boxed{R = \Sigma F_x}$$

$$= 2 + 4 - 5 - 8 = -7\text{ kN}$$

$\therefore R = 7\text{ kN} (\leftarrow)$

By Varignon's theorem, considering anticlockwise moments as positive about B and considering resultant (R) above B.

$$\boxed{\Sigma M_B^F = M_B^R}$$

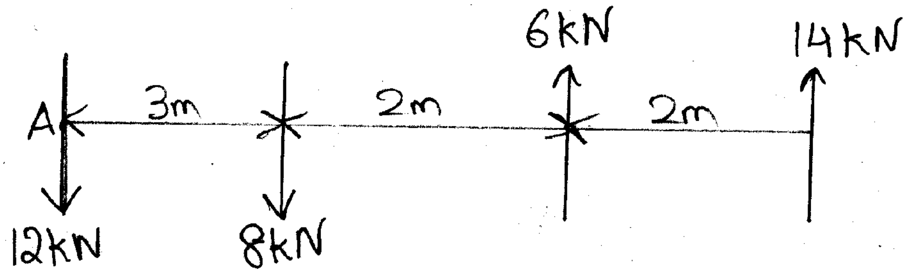
$$(8 \times 1.5) - (4 \times 3) - (2 \times 4) + (5 \times 6) = -(R \times d)$$

$$-22 = -7d$$

$$\therefore d = 3.143\text{m}$$

$\therefore$ Resultant force $\underline{R = 7\text{ kN} (\leftarrow)}$ at a perpendicular distance of $\underline{3.143\text{m}}$ above B.

Q2.11



$\boxed{R = \Sigma F_y}$

$= -12 - 8 + 6 + 14 = 0$

$\therefore R = 0$

Since resultant is zero, therefore a couple exists.

Considering moment about A, and taking anticlockwise moments as positive.

$\therefore \boxed{\Sigma M_A} = -8 \times 3 + 6 \times 5 + 14 \times 7$

$= 104 \text{ kNm}$.

$\therefore$ Resultant is a couple of <u>104 kNm</u> in anticlockwise direction.

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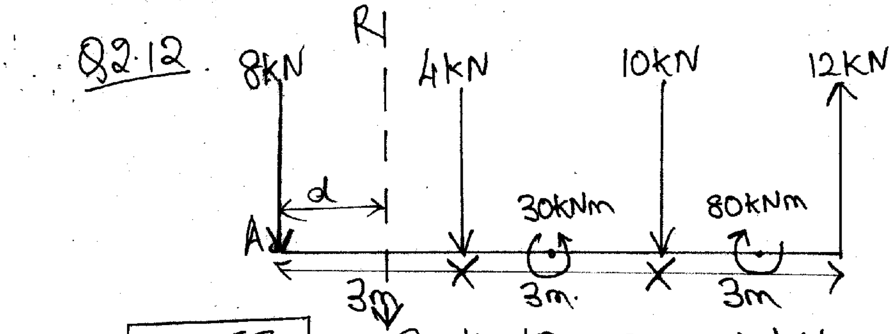
Q2.12



(i) $\boxed{R = \Sigma F_y} = -8 - 4 - 10 + 12 = -10\text{kN}$

$\therefore R = 10\text{kN} (\downarrow)$

By Varignon's theorem, considering anticlockwise moments to be positive and resultant to right of A

$\therefore \boxed{\Sigma M_A^F = M_A^R}$

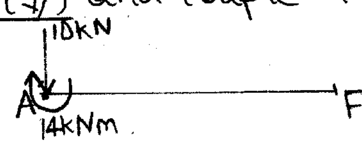
$\therefore (-4\times3) - (10\times6) + (12\times9) + 30 - 80 = -(R\times d)$

$\therefore -26 = -10\times d \quad \therefore d = 1.4\text{m}$.

$\therefore$ Resultant force $\underline{R = 10\text{kN}(\downarrow)}$ at a perpendicular distance of $\underline{1.4\text{m}}$ to the right of A.

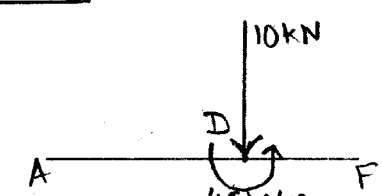
(ii) $\boxed{M_A^R = -R\times d} = -10\times1.4 = -14\text{kNm}$.

$\therefore$ Single force at A is $\underline{R = 10\text{kN}(\downarrow)}$ and couple is $\underline{14\text{kNm clockwise}}$.



(iii) $\boxed{M_D^R = R\times(6-d)} = 10\times4.6 = 46\text{kNm}$

$\therefore$ Single force at D is $\underline{R = 10\text{kN}(\downarrow)}$ and couple is $\underline{46\text{kNm anticlockwise}}$



(iv) $R_B + R_D = R$

$\therefore R_B + R_D = -10 \quad \text{---} (I)$

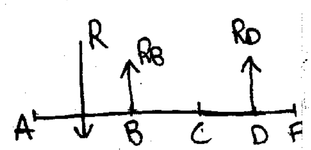
$M_B^F = M_B^R$

$\therefore R_D\times(3) = R\times(3-1.4)$

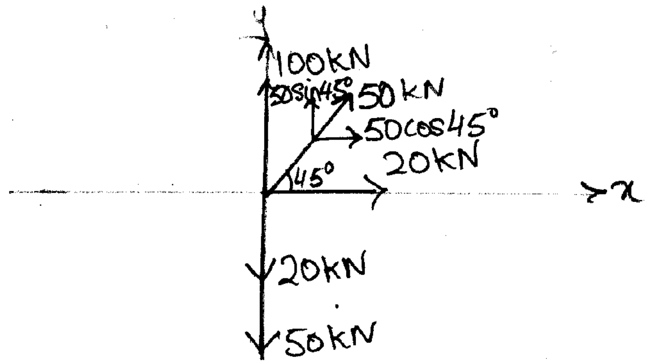
$\therefore 3R_D = 1.6(+10) \quad \therefore R_D = +5.33\text{kN} (\uparrow)$

Subs. in (I) $\therefore R_B = -15.33\text{kN}$

$\therefore R_B = 15.33\text{kN}(\downarrow)$



Q2.13



$$\Sigma F_x = 20 + 50\cos 45^\circ = 55.36 \text{ kN}$$

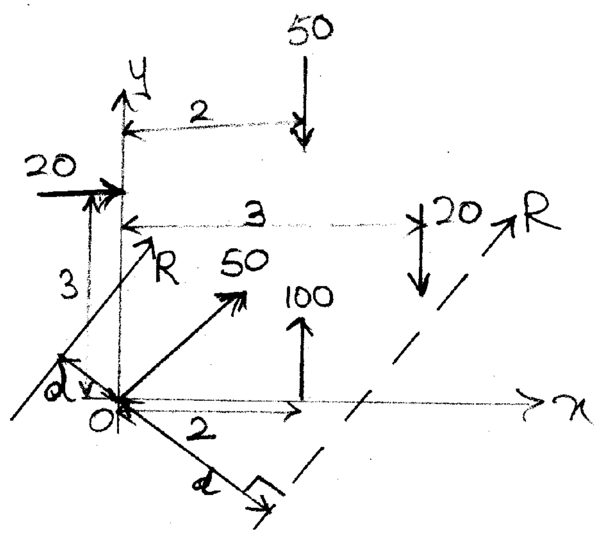
$$\Sigma F_y = 100 + 50\sin 45^\circ - 20 - 50 = 65.36 \text{ kN}$$

$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$

$$\therefore R = 85.65 \text{ kN}$$

$$\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}} = \frac{65.36}{55.36}$$

$$\therefore \theta = 49.74^\circ$$



By Varignon's theorem, considering anti-clockwise moments to be positive and resultant to right of O.

$$\boxed{\Sigma M_O^F = M_O^R}$$

$$(-20\times 3) + (100\times 2) - (50\times 2) - (20\times 3) = R\times d$$

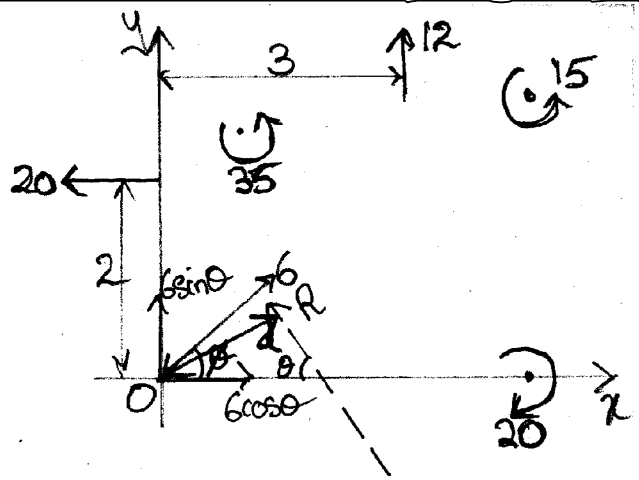
$$-20 = 85.65 \times d$$

$$\therefore d = -0.234 \text{ m}.$$

$\therefore$ Resultant force $\underline{R = 85.65\text{ kN}}$ at $\underline{\theta = 49.74^\circ}$ at a perpendicular distance of $\underline{0.234\text{ m}}$ to the left of O.

POWER

Q2.14



$\tan\phi = \frac{4}{5} \therefore \phi = 38.66°$

$\Sigma F_x = 6\cos\theta - 20$
$= -15.32\text{ N}$

$\Sigma F_y = 12 + 6\sin\theta$
$= 15.75\text{ N}$

$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$

$R = 21.96\text{ N}$; $\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}}$ $\therefore \theta = 45.79°$

By Varignon's theorem, considering anti-clockwise moments positive and resultant to the right of O.

$\boxed{\Sigma M_O^F = M_O^R}$

$(20\times2) + (12\times3) + 35 + 15 - 20 = (R\times d)$

$\therefore 106 = 21.96 \times d$

$\therefore d = 4.83\text{ m}$

$\boxed{\sin\theta = \frac{d}{x}}$ $\therefore x = \frac{4.83}{\sin(45.79°)} = 6.74\text{m}.$

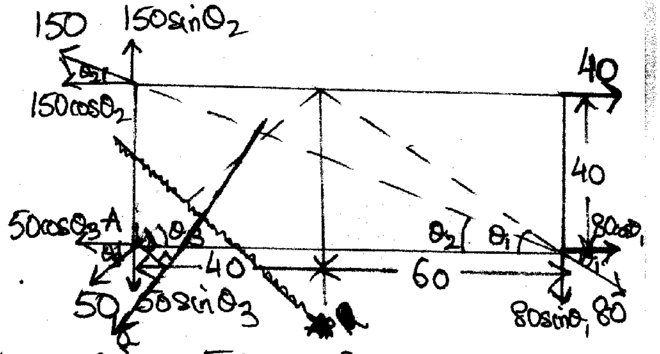
$\therefore$ Resultant force $\underline{R = 21.96\text{ N}}$ at $\underline{\theta = 45.79°}$ cuts the x-axis at $\underline{6.74\text{m}}$ to the right of O.

Q2.15

$\tan\theta_1 = \dfrac{40}{60} \quad \therefore \theta_1 = 33.69^\circ$

$\tan\theta_2 = \dfrac{40}{100} \quad \therefore \theta_2 = 21.8^\circ$

$\tan\theta_3 = \dfrac{40}{40} \quad \therefore \theta_3 = 45^\circ$



$\Sigma F_x = 40 + 80\cos\theta_1 - 150\cos\theta_2 - 50\cos\theta_3$

$= -68.06\ N$

$\Sigma F_y = 150\sin\theta_2 - 50\sin\theta_3 - 80\sin\theta_1$

$= -24.03\ N$

$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$

$= 72.18\ N$

$\boxed{\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x}} = \dfrac{24.03}{68.06}$

$\therefore \theta = 19.45^\circ$

By Varignon's theorem, considering anticlockwise moments to be positive and resultant to right of A

$\therefore \boxed{\Sigma M_A^F = M_A^R}$

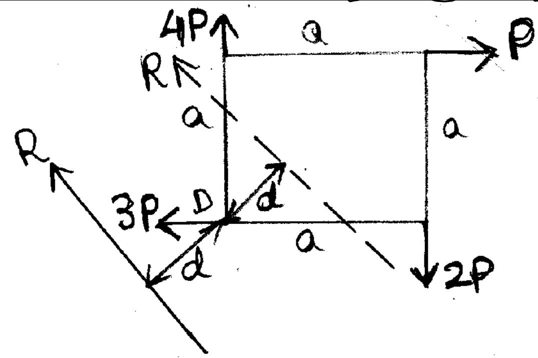
$(-40\times40) - (80\sin\theta_1 \times 100) + (150\cos\theta_2 \times 40) = R\times d$

$\therefore -466.68 = -72.18 \times d$

$\therefore d = 6.465\ cm$

$\therefore$ Resultant force is $\underline{R = 72.18\,N}$ at $\underline{\theta = 19.45^\circ}$ at a perpendicular distance of $\underline{6.465\,cm}$ to the right of A.

Q2.16



$\Sigma F_x = P - 3P = -2P$

$\Sigma F_y = 4P - 2P = 2P$

$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$

$R = 2.828P$

$\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}} = \frac{2P}{2P} = 1$

$\therefore \theta = 45°$

By Varignon's theorem, considering anticlockwise moments to be positive about D, and resultant to the right of D

$\boxed{\Sigma M_D^F = M_D^R}$

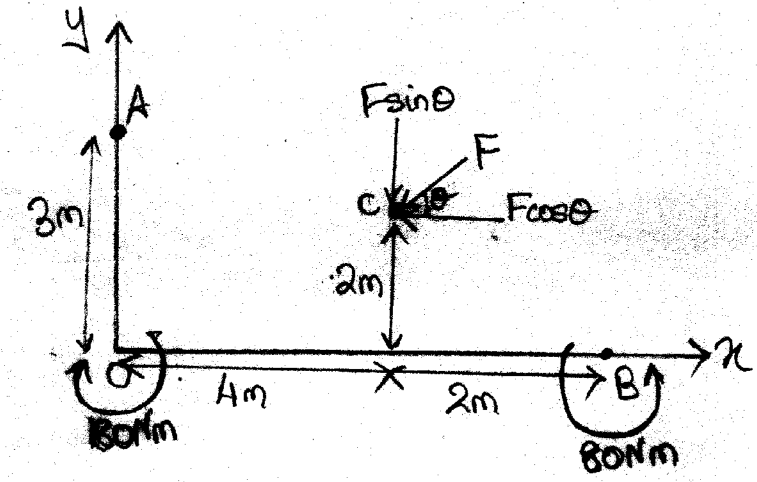
$(-P \times a) - (+2P \times a) = +R \times d$

$-3P \times a = +2.828P \times d$

$\therefore d = -1.06a$

$\therefore$ Resultant force is $\underline{R = 2.828P}$ at $\underline{\theta = 45°}$ at a perpendicular distance of $\underline{1.06a}$ to the left of D

Q2.17



Since moments at C is zero, therefore force F passes through C. Taking moments at O

$\boxed{M_O^F = -180}$, considering anticlockwise moments as positive

$\therefore 2 \times F\cos\theta - 4 \times F\sin\theta = -180$

$\therefore F\cos\theta - 2F\sin\theta = -90$ — (I)

$\boxed{M_B^F = 80}$

$\therefore 2 \times F\cos\theta + 2 \times F\sin\theta = 80$

$\therefore F\cos\theta + F\sin\theta = 40$ — (II)

(I) − (II)

$-3F\sin\theta = -130$

$\therefore F\sin\theta = 43.33$ — (III)

Subs. in (II)

$\therefore F\cos\theta = -3.33$ — (IV)

(III) ÷ (IV) $\therefore \tan\theta = 13.01$ $\therefore \theta = 85.6$.

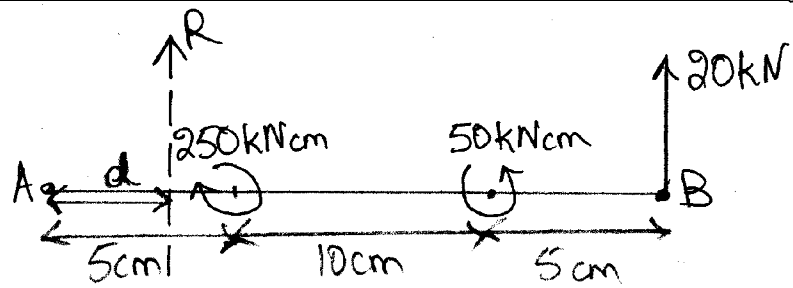
From (III), $F = 43.46$ N (In IV Quadrant $\because F\sin\theta$ is $\downarrow$ & $F\cos\theta \rightarrow$)

$M_A^F = -(1 \times F\cos\theta) - (4 \times F\sin\theta)$

$= -176.66$ Nm

$M_A^F = 176.66$ Nm clockwise.

Q2.18



$$\boxed{R = \Sigma F_y}$$

$R = 20kN$

By Varignon's theorem, considering anticlockwise moments to be positive about A and resultant force to the right of R.

$$\boxed{\Sigma M_A^F = M_A^R}$$

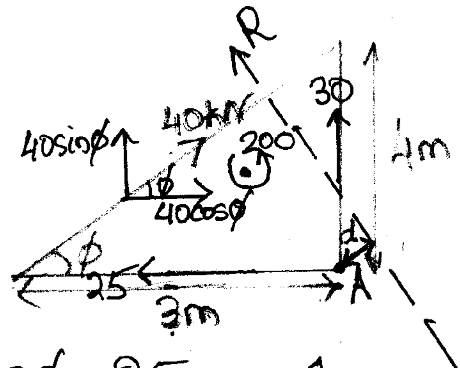
$\therefore +(20 \times 20) - 250 + 50 = R \times d$

$+200 = 20 \times d$

$\therefore d = 10cm.$

$\therefore$ Resultant force is <u>R = 20kN</u> at a distance of <u>10cm</u> to the right of A.

32.19



$\tan\phi = \frac{4}{3}$

$\therefore \phi = 53.13^\circ$

$\Sigma F_x = 40\cos\phi - 25 = -1$

$\Sigma F_y = 40\sin\phi + 30 = 62$

$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$

$\therefore R = 62.01$ kN

$$\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}} = 62$$

$\therefore \theta = 89.08^\circ$

By Varignon's theorem, considering anticlockwise moments to be positive about A and resultant force to the right of A.

$$\boxed{\Sigma M_A^F = M_A^R}$$

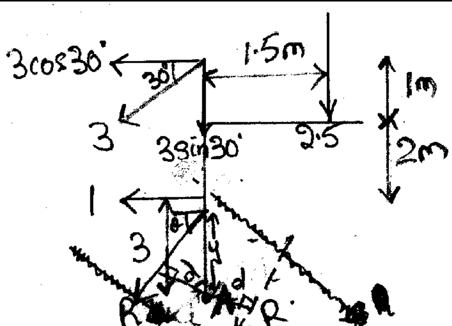
$\therefore -(3 \times 40\sin\phi) + 200 = R \times d$

$\therefore 80.02 = 62.01 \times d$

$\therefore d = \cancel{1.29}\ 1.67$ m

Resultant force is R = 62.01 kN at $\theta = 89.08^\circ$ at perpendicular distance of ~~1.29~~ 1.67 m to the right of A.

82.20



(i) $\Sigma F_x = -1 - 3\cos 30° = -3.6$

$\Sigma F_y = -2.5 - 3\sin 30° = -4$

$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$

$\therefore R = 5.38\ kN$

$$\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}} = 1.111$$

$\therefore \theta = 48.01°$

By Varignon's theorem, considering anticlockwise moments to be positive and resultant force to the right of A

$$\boxed{\Sigma M_A^F = M_A^R}$$

$$(6 \times 3\cos 30°) + (1 \times 3) - (2.5 \times 1.5) = R \times d$$

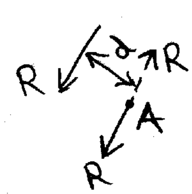
$$14.84 = -5.38 \times d$$

$\therefore d = -2.76\ m$. $\therefore d = 2.76\ m$.

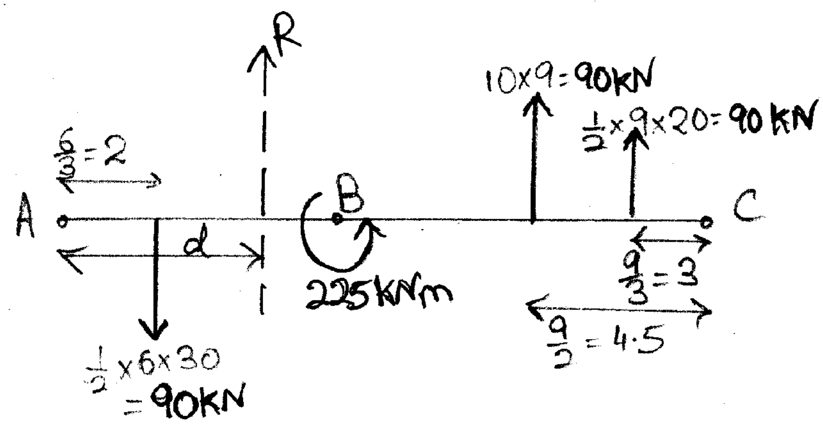
$\therefore$ Resultant force is $\underline{R = 5.38\ kN}$ at $\underline{\theta = 48.01°}$ at a perpendicular distance of $\underline{2.76m}$ to the left of A.

(ii) $\cos\theta = \frac{d}{y}$ $\therefore y = \frac{2.76}{\cos 48.01} = 4.126m$

(iii) $M_A^R = R \times d = 14.85\ kNm$ anticlockwise



Q2.21



$\boxed{R = \Sigma F_y} = 90 + 90 - 90 = 90\ kN$

By Varignon's theorem, considering anticlockwise moments to be positive about A and resultant force to the right of A.

$$\boxed{\Sigma M_A^F = M_A^R}$$

$$\therefore -(90 \times 2) + 225 + (90 \times 10.5) + (90 \times 12) = R \times d$$

$$2070 = 90 \times d$$

$$\therefore d = 23m.$$

$\therefore$ Resultant force is $\underline{R = 90kN (\uparrow)}$ at a perpendicular distance of $\underline{23m.}$ to the right of A.

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